1. Autograd. → Automatic Differentiation.

(a) $y = 3x^2 + 15$

$$\frac{dy}{dx} = 6x$$

$$\left.\frac{\partial y}{dx}\right|_{x=3} = 6 \times 3 = 18$$

(b) $z = 5x^3 + 7y^2$ (Partial Derivatives)

$$\frac{dz}{dx} = 15x^2 \qquad \left.\frac{dz}{dx}\right|_{x=3} = 15(3)^2$$

$$= 15 \times 9$$

$$= 135$$

$$\frac{dz}{dy} = 14y$$

$$\left.\frac{dz}{dy}\right|_{y=2} = 14 \times 2 = 28.$$

(c) Partial Derivatives.

$$y = x_1^3 + x_2^2 + 4x_1 x_2 + 5$$

$$\frac{\partial y}{dx_1} = 3x_1^2 + 4x_2 \Rightarrow \left.\frac{dy}{dx_1}\right|_{x_1=3,\, x_2=4}$$

$$= 3(3)^2 + 4(4) = 3\times 9 + 16$$

$$= 27 + 16 = 42$$

$$\frac{\partial y}{\partial x_2} = 2x_2 + 4x_1 \Rightarrow \left.\frac{\partial y}{\partial x_2}\right|_{x_1=3,\, x_2=4}$$

$$= 2(4) + 4(3) = 8 + 12 = 20.$$

(d) Multiple Calls to backward.

$$y = 3x^2 + 15$$

$$\frac{\partial y}{dx} = 6x$$

$$\underbrace{\left.\frac{\partial y}{dx}\right|_{x=3} = 6\times 3 = 18}_{\text{call : 1}}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = 6.}$$

$$\underbrace{\left.\frac{\partial y}{\partial x}\right|_{x=3} = 18 + 18}_{\text{call : 2}}$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=3} = 18 + 18 + 18$$

$$\underbrace{= 54}_{\text{call : 3?}}$$

Automatic Inclusion / Exclusion from DAG.

$$a = x + y$$

$$b = x + z$$

$a = x + 2 = [2, 3, 4]$

$i=0, i=1, i=2$

(d) Example → 4

$i{:}0, i=1, i=2$

$X = [0, 1, 2].$

$$a = x + 2$$

$$b = a^2$$

$$c = b + 3$$

$$y = c.\text{mean}().$$

x.grad → $\dfrac{\partial y}{\partial x_i} = \dfrac{\partial y}{\partial c_i} \cdot \dfrac{\partial c_i}{\partial b_i} \cdot \dfrac{\partial b_i}{\partial a_i} \cdot \dfrac{\partial a_i}{\partial x_i}$

$i \to i^{th}$ index.

H/W

$\dfrac{\partial a_i}{\partial x_i} = 1$ | $\dfrac{\partial b_i}{\partial a_i} = 2.a_i$ | $\dfrac{\partial c_i}{\partial b_i} = 1$ | $\dfrac{\partial y}{\partial c} = \dfrac{1}{3}$

$2 \cdot a \cdot$

$x = [0, 1, 2]$

$\boxed{\begin{array}{l} a = 0+2 \\ x = 0 \end{array}}$

$$\frac{\partial y}{\partial x_0} = \frac{1}{3} \times 1 \times (2 \times 2) \times 1 = \frac{4}{3}$$

$$\frac{\partial y}{\partial x_1} = \frac{1}{3} \times 1 \times (2 \times 3) \times 1 = 2$$

$\boxed{\begin{array}{l} x = 1 \\ a = x + 2 \\ \quad = 1 + 2 \\ \quad = 3 \end{array}}$

$$\frac{\partial y}{\partial x_2} = \frac{1}{3} \times 1 \times (2 \times 4) \times 1 = \frac{8}{3}$$

$x = 2$
$a = 2 + 2$
$= 4$

Given that we are indexing from 0, we get,

$$\Rightarrow \left[\frac{\partial y}{\partial x_0}, \frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2} \right] = \left[\frac{4}{3}, 2, \frac{8}{3} \right]$$

$\left[\frac{\partial y}{\partial x}\right]$, Are the gradients

(*).

Test example. 3-4 30-40

$$L = \text{loss} = (2x + y)^2$$

$$\frac{\partial L}{\partial x} = 2(2x+y) \cdot 2 = 4(2x+y)$$

$x \cdot \text{grad}()$.

$$= 8x + 4y$$

$$\frac{\partial L}{\partial y} = 2(2x+y) \cdot 1 = 4x + 2y.$$

index wise.

$\{ \quad x = [1, 2, 3] \qquad y = [1, 1, 1]$

$[8x_i + 4y_i] \rightarrow$ format

$$[8+4,\ 16+4,\ 24+4] = [12, 20, 28]$$

$[4x_i + 2y_i]$

$$y.grad = [4+2,\ 8+2,\ 12+2]$$

$$y.grad. = [6,\ 10,\ 14]$$

Multiple losses